NORMAL MODE vs PARABOLIC EQUATION AND THEIR APPLICATION IN TONKIN GULF

MODE CHUẨN SO VỚI PHƯƠNG TRÌNH PARABOLIC VÀ ÁP DỤNG VÀO VỊNH BẮC BỘ

Tran Cao Quyen

ABSTRACT

Normal Mode (NM) and Parabolic Equation (PE) have been used widely by Underwater Acoustic Community due to their effectiveness. In this paper, author investigates NM and PE in term of their mathematical approach as well as their computation. Further, Tonkin Gulf has been modeled and simulated using both of NM and PE. The simulation results show that there are the agreement and the reliability between both methodologies.

Keywords: *SONAR, Parabolic Equation, Normal Mode, Tonkin Gulf.*

TÓM TẮT

Phương pháp Mode chuẩn và phương trình Parabolic được dùng rộng rãi trong cộng đồng thủy âm vì sự hiệu quả của chúng. Trong bài báo này, tác giả nghiên cứu mode chuẩn và phương trình Parabolic ở khía cạnh toán học và tốc độ tính toán. Hơn nữa, Vịnh Bắc Bộ được mô hình hóa và mô phỏng dùng cả mode chuẩn và phương trình Parabolic. Các kết quả mô phỏng cho thấy có sự đồng nhất và tin cậy giữa hai phương pháp trên.

Từ khóa: *SONAR, Phương trình Parabolic, Mode chuẩn, Vịnh Bắc Bộ.*

Faculty of Electronics and Telecommunications,
VNU University of Engineering and Technology
Email: quyentc@vnu.edu.vn
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1. INTRODUCTION

First, sound propagation in ocean waveguide is investigated for a long time since its important role in SONAR (Sound navigation and ranging) techniques. As we known, there are numerous ways of the underwater sound modeling which appeared in time order namely ray, normal mode (NM) and parabolic equation (PE) [1].

Second, the NM is introduced the first time independently by Pekeris [2] and Ide [3] and then is classified by Williams [4]. After some decades of development of the NM, it becomes one of the most powerful approach of ocean acoustic computation. The best idea of NM is that it considers an acoustic pressure as an infinite number of modes which are similar to those obtained from a vibrating string. Each mode corresponds to an *eigenfunction* (mode shape) and an *eigenvalue* (horizontal propagation constant).

Third, the PE method is introduced firstly by Tappert [5] and is considered the modern method since it applied for the medium which has layers separated unclearly [5-8]. The advantages of parabolic method consists of using a source with one-way propagation, applying for range dependence, as well as performing in the medium which is not required exactly layered separation.

In this paper we investigate NM and PE in term of their mathematical approach as well as their computation. Besides, Tonkin Gulf has been modeled and simulated using not only NM but also PE. The obtained results show that when we divided the grid small enough (the depth, $\Delta z \le \frac{\lambda}{4}$, the range, $\Delta r = (5-10)\Delta z$, the parabolic algorithm converged fast. The achieved results of transmission loss factors (TLs) shows that there is a consistent agreement of TLs between NM and PE. The computation of PE is slightly more than NM.

The rest of the paper is organized as follows. Section 2 presents the mathematical representations of NM and the PE. We evaluate the NM and PE model in Tonkin gulf in section 3. Section 4 is our discussions. We conclude the paper in section 5.

2. NORMAL MODE AND PARABOLIC EQUATION

2.1. The Normal Mode

Staring from Helmholtz equation in two dimensions with sound speed *c* and density *ρ* depending only on depth z [1]:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \psi}{\partial r})+\rho(z)\frac{\partial}{\partial z}(\frac{1}{\rho(z)}\frac{\partial \psi}{\partial z})+\frac{\omega^2}{c(z)^2}\psi=-\frac{\delta(r)\delta(z-z_s)}{2\pi r} \quad (1)$$

where z_s is source depth, *z* is depth and *r* is distance.

Using separation of variables $\psi(r,z)=\Phi(r).V(z)$, we obtain the modal equation

$$\rho(z)\frac{d}{dz}[\frac{1}{\rho(z)}\frac{dV_m(z)}{dz}]+[\frac{\omega^2}{c(z)^2}-k_{rm}{}^2]V_m(z)=0 \quad (2)$$

with the boundary conditions such as

$$V(0)=0, \frac{dV}{dz}\Big|_{z=D}=0 \quad (3)$$

The former condition implies a pressure release surface and the latter condition is from a perfect rigid bottom. The modal equation that is the center of the NM, has an infinite number of modes. Each mode represents by a mode amplitude $V_m(z)$ and a horizontal propagation constant k_{rm}. $V_m(z)$ and k_{rm} are also called *eigenfunction* and *eigenvalue* respectively

Noting that the modes are orthonormal, i.e.,

$$\int_0^D \frac{V_m(z)V_n(z)}{\rho(z)}dz=0, \quad m\neq n$$
$$\int_0^D \frac{V_m(z)^2}{\rho(z)}dz=1 \quad (4)$$

Since the modes forms a complete set, the pressure can represents as a sum of the normal modes

$$\psi(r,z)=\sum_{m=1}^{\infty}\Phi_m(r)V_m(z) \quad (5)$$

After some manipulations, we obtain

$$\psi(r,z)=\frac{i}{4\rho(z_s)}V_m(z_s)H_0^1(k_{rm}r) \quad (6)$$

where H_0^1 is the Hankel function of the first kind.

Substitute (6) back to (5) we have

$$\psi(r,z)=\frac{i}{4\rho(z_s)}\sum_{m=1}^{\infty}V_m(z_s)V_m(z)H_0^1(k_{rm}r) \quad (7)$$

Finally, using the asymptotic approximation of the Hankel function, the pressure can be written as

$$\psi(r,z)\approx\frac{i}{\rho(z_s)\sqrt{8\pi r}}e^{-i\pi/4}\sum_{m=1}^{\infty}V_m(z_s)V_m(z)\frac{e^{ik_{rm}r}}{\sqrt{k_{rm}}} \quad (8)$$

2.2. The Parabolic Equation

Starting from the Helmholtz equation in the most general form [1]

$$\nabla^2\psi+k_0^2(n^2-1)\psi=0 \quad (9)$$

where *n* is the refraction index of the medium and k_0 is the wavenumber at the acoustic source.

In cylindrical coordinate, (1) becomes

$$\psi_{rr}+\frac{1}{r}\psi_r+\psi_{zz}+k_0^2(n^2-1)\psi=0 \quad (10)$$

in which the subscripts denote the order of derivative.

From the assumption of Tappert [5-6], ψ is defined as

$$\psi(r,z)=\Phi(r,z)V(r) \quad (11)$$

where z denotes depth and *r* denotes distance.

Thus (10) becomes the system of equations as follows

$$\Phi_{rr}+\left(\frac{1}{r}+\frac{2}{V}V_r\right)\Phi_r+\Phi_{zz}+k_0^2(n^2-1)\Phi=0 \text{ (4) and} \quad (12)$$

$$V_{rr}+\frac{1}{r}V_r+k_0^2V=0 \quad (13)$$

The root of (13) is a Hankel function with its approximation as

$$V_{r0}=H_0^1(k_0r)=\sqrt{\frac{2}{\pi k_0 r}}e^{i(k_0r-\frac{\pi}{4})} \quad (14)$$

After some manipulations, (12) becomes

$$2ik_0\Phi_r-\Phi_{zz}+k_0^2(n^2-1)\Phi=0 \quad (15),$$

i.e. a parabolic equation.

Taking the Fourier transform both side of (15) in z domain obtained

$$2ik_0\Phi_r-k_z^2\Phi+k_0^2(n^2-1)\Phi=0 \quad (16)$$

Rewrite (16) in simpler form as

$$\Phi_r+\frac{k_0^2(n^2-1)-k_z^2}{2ik_0}\Phi=0 \quad (17)$$

Thus, from [9] we have

$$\Phi(r,k_z)=\Phi(r_0,k_z)e^{\frac{-k_0^2(n^2-1)-k_z^2}{2ik_0}(r-r_0)} \quad (18)$$

where $\Phi(r_0,k_z)$ is the initial value of the source.

Taking the Inverse Fourier transform both side of (18) obtained

$$\Phi(r,z)=e^{i\frac{k_0}{2}(n^2-1)\Delta r}\int_{-\infty}^{\infty}\Phi(r_0,k_z)e^{\frac{-i\Delta rk_z^2}{2ik_0}}e^{ik_zz}dk_z \quad (19)$$

where $\Delta r=r-r_0$.

Finally, we arrived

$$\Phi(r,z)=e^{i\frac{k_0}{2}(n^2-1)\Delta r}\Im^{-1}\left\{e^{\frac{-i\Delta rk_z^2}{2ik_0}}\Im\{\Phi(r_0,z)\}\right\} \quad (20)$$

This form is called Split-Step Fourier transform.

3. SIMULATION RESULTS

3.1. The acoustic and noise source

The point source with the center frequency of 250Hz and the depth of 99m is used in this simulation. We assume that the receiver is placed at the same transmitter's depth; the noise source is Gaussian and the SNR level of 3dB.

3.2. Medium parameters

Table 1. The medium parameters

Paremeter	Value
Ocean depth	100m
Sound speed in winter	$c(z)=1500+0.3z$ (m/s)
Bottom	Sand, $\rho_1=2000$ kg/m^3 $c_1=1700$ m/s

In this simulation, Tonkin gulf is used as Pekeris waveguide model with its sound velocity which is measured from [10]. Thuc was carried out many sound speed measurements which were reported in his monograph. On the basis of Thuc's results, the medium parameters of Tolkin gulf are given in the Table 1.

In Table1, c denotes sound velocity whereas ρ indicates medium density.

3.3. Simulation Results

The transmission loss factors (TLs) of NM and PE are shown in Figure 1 and 2.

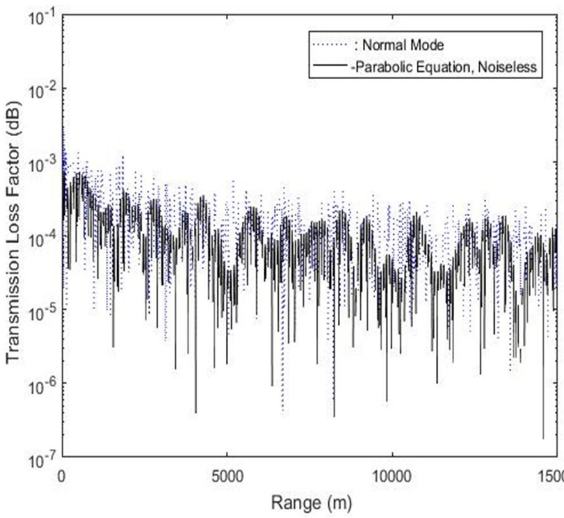

Figure 1. Transmission loss factors of NM and PE with range up to 15km, noiseless case

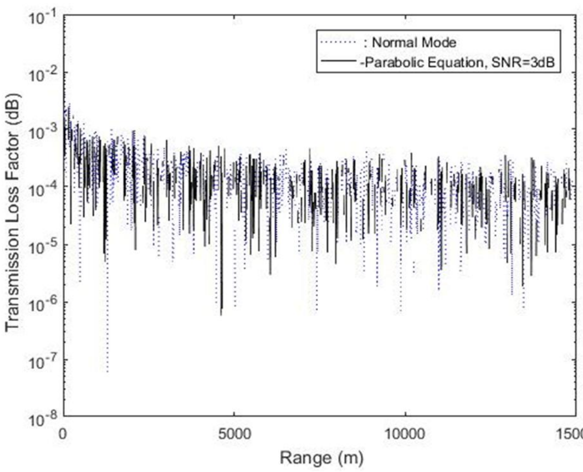

Figure 2. Transmission loss factors of NM and PE with range up to 15km, SNR = 3dB

4. DISCUSSIONS

From Figure 1 and Figure 2 we can see clearly that the TLs of both NM and PE with range up to 15km far from the acoustic source. In the conditions of this simulation, this TLs are stable after hundreds of simulations. Further, there is the agreement of TLs between NM and PE.

In the first case (noiseless case), from Figure 1, the TL of PE seems reducing to distance more slightly than the TL of NM. It is basically, could be thought of the nature of range dependence of PE approach.

In the second case (when SNR of 3dB), from Figure 2, the agreement of TLs of both methods is more consistent since the signal level in this case is higher than the noise level and it is compensated for a long range transmission.

The computation of PE is slightly more than NM (it is not shown here).

5. CONCLUSIONS

In this paper, the rigorous mathematical analyses of NM and PE are presented. The idea behind NM is vibrating of modes along depth axis and behind PE are one-way propagation and using Split-Step Fourier transform. Further, in conditions of this simulation, there is a consistent agreement of TLs between NM and PE in both noise and noiseless cases.

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THÔNG TIN TÁC GIẢ

Trần Cao Quyền

Khoa Điện tử - Viễn thông, Trường Đại học Công nghệ, Đại học Quốc gia Hà Nội